

Texture Zeros and Weak Basis Transformations

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Abstract

We investigate the physical meaning of some of the “texture zeros” which appear in most of the *Ansätze* on quark masses and mixings. It is shown that starting from arbitrary quark mass matrices and making a suitable weak basis transformation one can obtain some of these sets of zeros which therefore have no physical content. We then analyse the physical implications of a four-texture zero *Ansatz* which is in agreement with all present experimental data.

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I. INTRODUCTION

Understanding the structure of quark masses and mixings is one of the major open questions in particle physics. Recently, one has had increasingly more accurate experimental data [1] on the value of quark masses, as well as on the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. From this wealth of experimental data, one may be tempted to extract a clue about a symmetry principle, probably imposed at high-energy scale, which could lead to the pattern of quark masses and mixings, observed at low-energy scale. One of the difficulties one encounters in implementing this bottom-up approach results from the large redundancy contained in the quark mass matrices within the framework of the Standard Model (SM). In the SM, the flavour structure of Yukawa couplings is not constrained by gauge symmetry and, as a result, the up and down quark mass matrices are arbitrary complex matrices, thus containing a total of 36 free parameters. This number is to be compared to the ten physical parameters corresponding to the six quark masses and four physical parameters of the CKM matrix. The above redundancy is closely related to the fact that one has the freedom to make weak-basis (WB) transformations under which the quark mass matrices change but the gauge currents remain diagonal and real. Two sets of quark mass matrices related by a WB transformation obviously have the same physical content.

In the literature, various approaches to the flavour puzzle have been pursued, including the systematic search for texture zeros [3]. In view of the freedom in the choice of WB, it is important to analyse within the SM when does a set of zeros results only from a choice of WB and when it does imply restrictions among quark masses and/or mixings.

In this paper, we will analyse systematically the physical content of texture zeros within the framework of hermitian quark mass matrices. At this point, it is worth recalling that, within the SM, starting from arbitrary quark mass matrices \tilde{M}_u , \tilde{M}_d , one can always make a WB transformation under which $\tilde{M}_u \rightarrow M_u$, $\tilde{M}_d \rightarrow M_d$, with M_u , M_d hermitian matrices. Therefore, one does not lose generality by restricting the analysis to hermitian quark mass matrices.

We will consider some of the texture zeros which appear in most of the *Ansätze* proposed in the literature [3–10], namely those occurring in the elements (1,1), (1,3), (3,1) of the up and down quark mass matrices. We will show which combinations of texture zeros can be obtained from arbitrary quark mass matrices by simply making appropriate WB transformations. Zeros satisfying the above condition will be called WB zeros. Of course, these sets of WB zeros do not have any physical content by themselves. However, we will point out that some of these sets, when supplemented by some mild assumptions on the hierarchy of quark masses and mixings, do lead to physical predictions.

II. ZEROS ARISING FROM WB TRANSFORMATIONS

We will consider WB zeros occurring in the elements (1,1), (1,3) of the up and down quark mass matrices. Since we restrict ourselves to hermitian mass matrices, the vanishing of the (1,3) element obviously implies the vanishing of the (3,1) element. Taking into account that there are ten physical parameters in M_u , M_d , it is clear that there is a limit to the number of WB zeros which can be obtained, since the resulting M_u , M_d matrices have to contain at least ten independent parameters. Matrices with less than ten parameters lead to relations

between quark masses and/or mixings and therefore cannot reflect only a choice of WB. In view of this, we need to consider hermitian matrices containing at most four WB zeros, corresponding to ten physical parameters. We will first consider matrices with WB zeros in the (1,1) element of the up and down quark mass matrices and then study the simultaneous presence of WB zeros in the (1,1) and (1,3) elements.

A. The (1,1) WB zero

The most general WB transformation that leaves the mass matrices hermitian is:

$$\begin{aligned} M_u &\longrightarrow M'_u = W^\dagger M_u W, \\ M_d &\longrightarrow M'_d = W^\dagger M_d W, \end{aligned} \quad (1)$$

where W is an arbitrary unitary matrix. In such basis, the quark mass matrices can be diagonalized by the set of unitary matrices $\{U_u, U_d\}$ such that

$$\begin{aligned} D_u &= U_u^\dagger M_u U_u, \\ D_d &= U_d^\dagger M_d U_d, \end{aligned} \quad (2)$$

where $D_u \equiv \text{diag}(m_u, m_c, m_t)$ and $D_d \equiv \text{diag}(m_d, m_s, m_b)$. We emphasize that the relative sign of the quark mass parameters m_i ($i = u, c, t, d, s, b$) does not have physical meaning since it can always be changed by a WB transformation.

Let us start by choosing a WB where the up quark mass matrix M_u is diagonal and the down quark mass matrix M_d is hermitian, i.e.

$$\begin{aligned} M_u &= D_u, \\ M_d &= V D_d V^\dagger. \end{aligned} \quad (3)$$

The matrix V is an arbitrary unitary matrix, which in a physical context would correspond to the usual CKM matrix.

Since we are interested in a WB where the (1,1)-matrix elements of the quark mass matrices vanish, let us make a WB transformation W under which M_u, M_d transform as:

$$\begin{aligned} M_u &\longrightarrow M'_u = W^\dagger D_u W, \\ M_d &\longrightarrow M'_d = W^\dagger V D_d V^\dagger W, \end{aligned} \quad (4)$$

such that $(M'_u)_{11} = (M'_d)_{11} = 0$. This requires the solution of the system of equations

$$\begin{aligned} m_u |W_{11}|^2 + m_c |W_{21}|^2 + m_t |W_{31}|^2 &= 0, \\ m_d |X_{11}|^2 + m_s |X_{21}|^2 + m_b |X_{31}|^2 &= 0, \\ |W_{11}|^2 + |W_{21}|^2 + |W_{31}|^2 &= 1, \end{aligned} \quad (5)$$

where $X \equiv V^\dagger W$, and thus:

$$\begin{aligned} |X_{i1}|^2 &= |V_{1i}|^2 |W_{11}|^2 + |V_{2i}|^2 |W_{21}|^2 + |V_{3i}|^2 |W_{31}|^2 \\ &\quad + 2 \text{Re}(V_{1i}^* W_{11} V_{2i} W_{21}^*) + 2 \text{Re}(V_{1i}^* W_{11} V_{3i} W_{31}^*) \\ &\quad + 2 \text{Re}(V_{2i}^* W_{21} V_{3i} W_{31}^*), \quad (i = 1, 2, 3). \end{aligned} \quad (6)$$

Notice that for the system (5) to have a real solution, it is necessary that at least one of the mass parameters m_u, m_c, m_t and one of the parameters m_d, m_s, m_b be negative.

Given arbitrary matrices M_u, M_d or equivalently D_u, D_d, V , one has to find a unitary matrix W satisfying (5). In general, it is not an easy task to find a closed analytical solution for the system of Eqs.(5). In section III we will give some numerical examples. Next, we shall study some special cases where a simple analytical solution for W can be found.

Let us consider first the special case of $V = \mathbb{1}$. In this case, $X = W$ and the solution of the system of equations (5) is straightforward:

$$\begin{aligned} |W_{11}|^2 &= \frac{m_c m_b - m_s m_t}{\Delta}, \\ |W_{21}|^2 &= \frac{m_d m_t - m_u m_b}{\Delta}, \\ |W_{31}|^2 &= \frac{m_u m_s - m_d m_c}{\Delta}, \end{aligned} \quad (7)$$

where

$$\Delta = (m_t - m_u)(m_b - m_s) - (m_t - m_c)(m_b - m_d). \quad (8)$$

Next we consider another example, where it is also possible to find an analytical solution for W . We choose V closer to a realistic CKM matrix:

$$V = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

In this case, Eqs.(6) lead to:

$$\begin{aligned} |X_{11}|^2 &= \cos^2 \theta |W_{11}|^2 + \sin^2 \theta |W_{21}|^2 - \sin 2\theta W_{11} W_{21}, \\ |X_{21}|^2 &= \sin^2 \theta |W_{11}|^2 + \cos^2 \theta |W_{21}|^2 + \sin 2\theta W_{11} W_{21}, \\ |X_{31}|^2 &= |W_{31}|^2, \end{aligned} \quad (10)$$

where we have assumed W_{i1} ($i = 1, 2, 3$) to be real.

Using unitarity, we can write

$$\begin{aligned} (m_u - m_t)|W_{11}|^2 + (m_c - m_t)|W_{21}|^2 + m_t &= 0, \\ (m_d \cos^2 \theta + m_s \sin^2 \theta - m_b)|W_{11}|^2 \\ + (m_d \sin^2 \theta + m_s \cos^2 \theta - m_b)|W_{21}|^2 \\ + (m_s - m_d) \sin 2\theta W_{11} W_{21} + m_b &= 0. \end{aligned} \quad (11)$$

The solution of the above system can be parametrized as:

$$\begin{aligned} \sqrt{m_t - m_u} W_{11} &= \sqrt{m_t} \cos \varphi, \\ \sqrt{m_t - m_c} W_{21} &= \sqrt{m_t} \sin \varphi. \end{aligned} \quad (12)$$

Denoting

$$\begin{aligned}
a &= m_b - (m_b - m_d \sin^2 \theta - m_s \cos^2 \theta) \frac{m_t}{m_t - m_c}, \\
b &= (m_s - m_d) \frac{m_t \sin 2\theta}{\sqrt{(m_t - m_u)(m_t - m_c)}}, \\
c &= m_b - (m_b - m_d \cos^2 \theta - m_s \sin^2 \theta) \frac{m_t}{m_t - m_u},
\end{aligned} \tag{13}$$

and introducing $z \equiv \tan \varphi$, the solution is simply given by the quadratic equation

$$a z^2 + b z + c = 0. \tag{14}$$

In particular for $\theta = 0$, i.e. $V = \mathbb{1}$, we recover the result of Eqs.(7).

In order to emphasize that one can obtain the (1,1) zero simultaneously in M'_u , M'_d , starting from arbitrary M_u , M_d , we consider next the case of a completely unrealistic V_{CKM} , namely:

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & \sin \theta & -\cos \theta \end{pmatrix}. \tag{15}$$

It can be readily shown that the solution of the system (5) can be obtained from the previous case by the substitutions $m_d \longleftrightarrow m_b$, $|W_{11}|^2 \longleftrightarrow |W_{31}|^2$, $m_u \longleftrightarrow m_t$ in Eqs.(11) and the corresponding changes in Eqs.(12)-(14).

B. The (1,3) WB zero

We have seen that it is possible in general to perform a WB transformation on the quark mass matrices such that the matrix elements in the position (1,1) are equal to zero. A natural question to ask is whether one can get additional WB zeros, besides the ones already obtained for the (1,1) matrix elements. In order to show that this is indeed possible, let us assume that one has already performed a WB transformation so that $(M_u)_{11} = (M_d)_{11} = 0$. It can be readily seen that there exists a second WB transformation that keeps $(M'_u)_{11} = (M'_d)_{11} = 0$ and leads to $(M'_d)_{13} = (M'_d)_{31} = 0$. Such a transformation is defined by Eqs.(4), where

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -e^{i\varphi} \sin \theta \\ 0 & e^{-i\varphi} \sin \theta & \cos \theta \end{pmatrix}, \tag{16}$$

with θ and φ given by

$$\tan \theta = \left| \frac{(M_d)_{13}}{(M_d)_{12}} \right|, \quad \varphi = \arg(M_d)_{13} - \arg(M_d)_{12}. \tag{17}$$

Once this WB transformation is performed, we are left with the following WB zero forms:

$$M'_u = \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{pmatrix}, \quad M'_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \quad (18)$$

At this point it is worth mentioning that the construction of mass matrices with two off-diagonal zeros in one matrix and one off-diagonal zero in the other matrix is straightforward, if one starts in the basis where one of the matrices is diagonal (cf. Eq.(3)) and then perform the WB given in (16). All the cases with two off-diagonal zeros in one sector and one off-diagonal zero in the other can be studied following this procedure. We are however concerned with the more interesting case when the zeros are also present in the diagonal matrix elements.

Next we address the question whether it is possible to obtain an additional zero at positions (1,3) and (3,1) of the up quark mass matrix leading to the following structure

$$M''_u = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad M''_d = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \quad (19)$$

The structure of the mass matrices given by Eq.(19) has the interesting feature of exhibiting a parallel structure for the up and down mass matrices and having ten independent parameters. This can be seen by noting that one can make quark phase redefinitions which render M''_u real, while leaving M''_d with two phases. Thus the number of independent parameters in M''_u , M''_d coincides with the number of physical parameters contained in the up and down quark mass matrices. Therefore, it is pertinent to ask whether one can always reach the form of Eq.(19), starting from arbitrary matrices M_u , M_d , through a WB transformation. It can be easily shown that this is generally not the case.

Let us assume that one starts from the basis of Eq.(3) and take the special case $V = \mathbb{1}$. Let us consider a WB transformation performed by a unitary matrix W , as defined by Eq.(4). In order for this WB transformation to lead to the structure of Eq.(19), W will have to satisfy not only Eqs.(7), but also the relations:

$$\begin{aligned} (m_t - m_u)W_{11}^*W_{13} + (m_t - m_c)W_{21}^*W_{23} &= 0, \\ (m_b - m_d)W_{11}^*W_{13} + (m_b - m_s)W_{21}^*W_{23} &= 0. \end{aligned} \quad (20)$$

It is clear that there is no unitary matrix W satisfying the above relations for arbitrary quark masses. Therefore the form of Eq.(19) does not reflect, in general, a WB choice; on the contrary, it implies constraints on quark masses and mixings. Nevertheless, there are CKM matrices and quark masses consistent with the present experimental data, for which it does exist a WB transformation leading to four WB zeros as in Eq.(19). In the next section we will present a numerical example where the matrix W is explicitly given.

III. NUMERICAL EXAMPLES

To present our numerical examples we shall use the standard parametrization of the CKM matrix advocated in [1],

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \quad (21)$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, $i, j = 1, 2, 3$. Since c_{13} is known experimentally to be very close to unity [1], then one has $V_{us} \simeq s_{12}$, $V_{ub} \simeq s_{13}$, $V_{cb} \simeq s_{23}$.

We shall take the following quark mass values (in GeV) given at the electroweak ($\mu = M_Z$) scale:

$$\begin{aligned} |m_u| &= 0.0025, & |m_c| &= 0.6, & |m_t| &= 174, \\ |m_d| &= 0.004, & |m_s| &= 0.08, & |m_b| &= 3. \end{aligned} \quad (22)$$

Let us consider first a CKM matrix consistent with the present experimental data. Assuming $V_{us} = 0.22$, $V_{ub} = 0.0036$, $V_{cb} = 0.04$ and $\delta_{13} = \pi/2$, we find from (21) that

$$|V| = \begin{pmatrix} 0.9755 & 0.22 & 0.0036 \\ 0.2198 & 0.9747 & 0.04 \\ 0.0095 & 0.039 & 0.9992 \end{pmatrix}. \quad (23)$$

Now it is easy to find the WB transformation (4) such that the new quark mass matrices contain four WB zeros at positions (1,1) and (1,3). Taking $m_u < 0$, $m_d < 0$ in (22), one has

$$W = \begin{pmatrix} -0.9979 & -0.0096 - 0.043i & -0.0459 + 0.0029i \\ 0.0137 - 0.0628i & -0.6946 & -0.1076 + 0.7084i \\ 0.0002 & -0.1078 - 0.7099i & 0.696 \end{pmatrix}, \quad (24)$$

and the quark mass matrices are

$$M'_u = \begin{pmatrix} 0 & -0.0118 - 0.0543i & 0 \\ -0.0118 + 0.0543i & 89.99 & -13.0092 + 85.6774i \\ 0 & -13.0092 - 85.6774i & 84.6076 \end{pmatrix}, \quad (25)$$

$$M'_d = \begin{pmatrix} 0 & 0.0242 - 0.0079i & 0 \\ 0.0242 + 0.0079i & 1.6007 & -0.3319 + 1.4223i \\ 0 & -0.3319 - 1.4223i & 1.4753 \end{pmatrix}. \quad (26)$$

In order to illustrate the fact that the form of Eq.(18) only reflects a choice of WB, we give next an example where we take the same realistic values for quark masses given by Eq.(22), but choose a completely unrealistic CKM matrix. Taking $\theta_{12} = \theta_{13} = \theta_{23} = \pi/6$ and $\delta_{13} = \pi/2$ in (21) we obtain:

$$|V| = \begin{pmatrix} 3/4 & \sqrt{3}/4 & 1/2 \\ \sqrt{15}/8 & \sqrt{37}/8 & \sqrt{3}/4 \\ \sqrt{13}/8 & \sqrt{15}/8 & 3/4 \end{pmatrix}. \quad (27)$$

It is easy to construct a unitary matrix W such that the WB transformation given in Eq.(4) transforms the quark mass matrices into the WB zero form of Eq.(18). Assuming for this case $m_c < 0, m_s < 0$ in (22) we find:

$$W = \begin{pmatrix} -0.7345 & 0.1049 - 0.4481i & -0.4986i \\ 0.6774i & 0.5272 + 0.1134i & 0.5002 + 0.009i \\ -0.0397i & 0.7052 - 0.0066i & -0.6908 + 0.1544i \end{pmatrix}, \quad (28)$$

$$M'_u = \begin{pmatrix} 0 & -0.0004 + 5.0843i & -1.0697 - 4.5653i \\ -0.0004 - 5.0843i & 86.373 & -85.1037 + 18.1793i \\ -1.0697 + 4.5653i & -85.1037 - 18.1793i & 87.0295 \end{pmatrix}, \quad (29)$$

$$M'_d = \begin{pmatrix} 0 & -0.0309 + 0.314i & 0 \\ -0.0309 - 0.314i & 2.9144 & -0.1218 + 0.3768i \\ 0 & -0.1218 - 0.3768i & 0.0096 \end{pmatrix}. \quad (30)$$

IV. FOUR TEXTURE ZEROS AND THE CKM MATRIX

The four texture zero form of Eq.(19) is specially interesting because the latest low energy data [1] seems to rule out all six and five texture zero quark mass matrices. Let us assume the following up and down quark mass matrices:

$$M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u & c_u \\ 0 & c_u & d_u \end{pmatrix}, \quad M_d = P \begin{pmatrix} 0 & a_d & 0 \\ a_d & b_d & c_d \\ 0 & c_d & d_d \end{pmatrix} P^\dagger, \quad (31)$$

where $P = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1)$. The matrix elements of a real matrix of the type

$$M = \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & d \end{pmatrix}, \quad (32)$$

are related to the eigenvalues m_i , $i = 1, 2, 3$, through the invariants:

$$\begin{aligned} \text{Tr}(M) &= b + d = m_1 + m_2 + m_3, \\ \det(M) &= -a^2d = m_1m_2m_3, \\ \chi(M) &= bd - a^2 - c^2 = m_1m_2 + m_1m_3 + m_2m_3. \end{aligned} \quad (33)$$

Without loss of generality we can order the eigenvalues m_i in such a way that $|m_1| < |m_2| < |m_3|$ and also assume that $m_1 < 0$, $m_3 > d > m_2 > 0$. In this case the unitary matrix U which diagonalizes M is given by

$$U = \begin{pmatrix} \sqrt{\frac{m_2m_3(d-m_1)}{d(m_2-m_1)(m_3-m_1)}} & \sqrt{\frac{m_1m_3(m_2-d)}{d(m_2-m_1)(m_3-m_2)}} & \sqrt{\frac{m_1m_2(d-m_3)}{d(m_3-m_1)(m_3-m_2)}} \\ -\sqrt{\frac{m_1(m_1-d)}{(m_2-m_1)(m_3-m_1)}} & \sqrt{\frac{(d-m_2)m_2}{(m_2-m_1)(m_3-m_2)}} & \sqrt{\frac{m_3(m_3-d)}{(m_3-m_1)(m_3-m_2)}} \\ \sqrt{\frac{m_1(d-m_2)(d-m_3)}{d(m_2-m_1)(m_3-m_1)}} & -\sqrt{\frac{m_2(d-m_1)(m_3-d)}{d(m_2-m_1)(m_3-m_2)}} & \sqrt{\frac{m_3(d-m_1)(d-m_2)}{d(m_3-m_1)(m_3-m_2)}} \end{pmatrix}. \quad (34)$$

If we assume the mass hierarchy $|m_1| \ll m_2 \ll m_3$, then the matrix U can be simplified:

$$U \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_1(m_2-d)}{d m_2}} & \sqrt{\frac{m_1 m_2 (d-m_3)}{d m_3^2}} \\ -\sqrt{\frac{d|m_1|}{m_2 m_3}} & \sqrt{\frac{d-m_2}{m_3}} & \sqrt{\frac{m_3-d}{m_3}} \\ \sqrt{\frac{m_1(d-m_2)(d-m_3)}{d m_2 m_3}} & -\sqrt{\frac{m_3-d}{m_3}} & \sqrt{\frac{d-m_2}{m_3}} \end{pmatrix}, \quad (35)$$

If we make the additional assumption that $d \approx m_3$ then the matrix U is even further simplified:

$$U \approx \begin{pmatrix} 1 & \sqrt{\frac{|m_1|}{m_2}} & \sqrt{\frac{m_1 m_2 (d-m_3)}{m_3^3}} \\ -\sqrt{\frac{|m_1|}{m_2}} & 1 & \sqrt{\frac{m_3-d}{m_3}} \\ \sqrt{\frac{m_1(d-m_3)}{m_2 m_3}} & -\sqrt{\frac{m_3-d}{m_3}} & 1 \end{pmatrix}, \quad (36)$$

Now one can easily find the CKM matrix $V = U_u^\dagger P U_d$. In particular, using the matrix form (36) for U_u, U_d we obtain the successful prediction [4],

$$|V_{us}| \simeq \left| \sqrt{\frac{|m_d|}{m_s}} - \sqrt{\frac{|m_u|}{m_c}} e^{i(\alpha_2 - \alpha_1)} \right|. \quad (37)$$

Moreover the following ratios are obtained:

$$\frac{|V_{ub}|}{|V_{cb}|} \simeq \sqrt{\frac{|m_u|}{m_c}}, \quad \frac{|V_{td}|}{|V_{ts}|} \simeq \sqrt{\frac{|m_d|}{m_s}}, \quad (38)$$

which are common to some models with nearest-neighbour mixing [8,11].

V. DISCUSSION AND CONCLUSIONS

In this letter we address the question of the physical meaning of texture zeros in the up and down quark mass matrices. We have pointed out that some of the zeros included in most of the *Ansätze* proposed in the literature only reflect a choice of WB, in the sense that one can obtain them starting from arbitrary mass matrices M_u, M_d , by making an appropriate WB transformation. This is the case of the three WB zero structure of Eq.(18). We have then examined the four texture zero structure of Eq.(19), which is specially important in view of the fact that the latest low energy data disfavors [12,13] all six and five texture zero quark mass matrices. We have also pointed out that not all matrices M_u, M_d can be put in the form of the Eq.(19), through a choice of WB. Nevertheless, we have shown there are cases of quark masses and mixings for which such a WB transformation indeed exists. This four texture zero structure has ten independent parameters and thus one would expect that it would have a rather limit predictive power, since there are also ten physical parameters

corresponding to the six quark masses and four CKM parameters. However, as we have seen in section IV, the four texture zero *Ansatz* of Eq.(19), together with some assumptions which include the quark mass hierarchies, does lead to successful predictions for V_{CKM} such as those of Eqs.(37),(38). Finally, it should be mentioned that another attractive feature of the four texture zero *Ansatz* considered here is the fact that it can successfully describe not only the quark but also the lepton sector, in particular the charged lepton and neutrino masses [14]. If the Dirac neutrino and Majorana mass matrices, M_D, M_R , are assumed to have a zero texture structure similar to the quark mass matrices, i.e. with zeros in positions (1,1),(1,3) and (3,1), then the neutrino mass matrix obtained via the see-saw mechanism, $M_\nu = -M_D^T M_R^{-1} M_D$, will have the same structure since the latter transformation does not affect the above zero textures.

Throughout this letter, we have used the fact that in the SM all WB are equivalent. In theories beyond the SM, where family symmetries may be present, not all WB are equivalent, since the symmetry may single out a particular WB. Note however that even if one finds a family symmetry which leads automatically to the four-texture zero structure of Eq.(19), in order to gain predictive power one has to find an additional mechanism to explain the mass hierarchies. An especially attractive mechanism is the one suggested by Froggatt and Nielsen [15] through a broken continuous Abelian symmetry beyond the SM.

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